

# Fractionalization into merons in quantum dots

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We study by exact diagonalization, in the lowest Landau level approximation, the Coulomb interaction problem of  $N = 4$  and  $N = 6$  quantum dot in the limit of zero Zeeman coupling. We find that meron excitations constitute the lowest lying states of the quantum dots. This is based on a mapping between the excitations of the dot and states of the Haldane-Shastry spin chain.

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*Introduction* Quasiparticles with fractional charge and fractional statistics make a hallmark of the fractional quantum Hall (QH) effect [1, 2]. They are usually found in spin polarized systems, but they can be found also in the systems where Zeeman energy is negligible and spin degree of freedom plays a role. In these systems quasiparticles, which carry charge, can be identified with topological objects, special configurations of spin in space. An example of this is skyrmion, the topological object well known from classical ferromagnetism [3], which is also a quasiparticle in the  $\nu = 1$  QH system [4]. We will show in this letter that as the size of the  $\nu = 1$  system shrinks to quantum dot (QD) another quasiparticles of a topological nature, merons [5], enter the stage and constitute the lowest lying excitations.

Each meron is a half of skyrmion [6]. If in the center of skyrmion excitation in 2D plane spin points up it slowly tumbles down to the down configuration on its circular boundary. On the other hand in the case of meron spin does not reach the down position on its boundary but it is half way between the up and down position i.e. it is in the plane and winds for  $2\pi$  along its boundary; see Fig. 1.

It is hard to prove existence of a quasiparticle in a finite, small system. In order to prove the existence of merons in QDs we used a mapping between seemingly uncorrelated physical systems, 2D QD with zero Zeeman coupling and 1D Haldane-Shastry (HS) spin chain [7]. It is not utterly suprising that it exists and maps the physics of spinons, excitations, of the HS chain to the

one of merons in QD. It is well known that the edge of QH systems can be mapped to one-dimensional models (charge excitations to Calogero-Sutherland model [8]) and this may exist even in small systems like QDs where the distinction between the bulk and edge is blurred.

*Model* We model a quantum dot in the regime of high magnetic fields in the lowest Landau level (LLL) approximation. Accordingly the Hamiltonian takes the following form,

$$H = H_{sp} + H_{int}, \quad (1)$$

where  $H_{sp}$  denotes the single particle part, without the Zeeman term in our case,  $H_{sp} = \hbar[\omega N + (\omega - \frac{1}{2}\omega_c)L]$ , where  $\omega_c = \frac{eB}{m^*}$ , the cyclotron frequency,  $\omega = \sqrt{\omega_o^2 + \frac{\omega_c^2}{4}}$  where  $\omega_o$  is the frequency of the harmonic confining potential,  $L$  and  $N$ , the total orbital angular momentum and number of particles of the dot respectively, and  $H_{int}$  denotes the interaction part,

$$H_{int} = \frac{1}{2} \sum a_{m_1\sigma}^\dagger a_{m_2\sigma'}^\dagger a_{m_3\sigma} a_{m_4\sigma'} \langle m_1 | \langle m_2 | V_{12} | m_4 \rangle | m_3 \rangle, \quad (2)$$

where  $a_{m\sigma}^\dagger$  and  $a_{m\sigma}$  create and annihilate electron with the spin projection  $\sigma$  in the single particle state of the LLL with angular momentum  $m \geq 0$ ,

$$\langle \mathbf{r} | m \rangle = \frac{1}{\sqrt{2\pi 2^m m!}} r^m \exp\{-im\phi\} \exp\{-\frac{r^2}{4}\}, \quad (3)$$

where  $\hbar = 1$  and  $2m^*\omega = 1$ .  $V_{12}$  is the Coulomb interaction operator,  $V_{12} = \frac{e^2}{4\pi\epsilon} \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|}$ , between two particles. As  $H_{sp}$  is trivially diagonalized and accounted for, we will refer in the following to the energies of  $H_{int}$  as those of  $H$ .

*HS spin chain* The HS Hamiltonian is

$$H_{HS} = J \left(\frac{2\pi}{N}\right)^2 \sum_{\alpha < \beta}^N \frac{\vec{S}_\alpha \vec{S}_\beta}{|z_\alpha - z_\beta|^2}, \quad (4)$$

where sites  $\alpha, \beta = 1, \dots, N$  are positioned on a unit circle so that each site coordinate,  $z_\alpha$ , fulfills  $z_\alpha^N - 1 = 0$ , and  $\vec{S}_\alpha$ 's are spin  $1/2$  operators. Any state of the chain can be

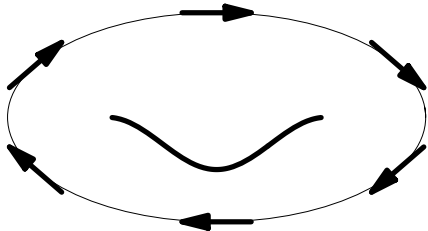


FIG. 1: Meron quasi-hole.

represented as a function of complex numbers satisfying  $z_\alpha^N = 1$ , and representing sites for which  $S_{\alpha z} = +1/2$ . In this way the ground state wave function, a spin singlet, is a function of  $N/2$  complex numbers,  $\Psi(z_1, \dots, z_{N/2}) = \prod_{j < k}^{N/2} (z_j - z_k)^2 \prod_{j=1}^{N/2} z_j$ . Spinons, elementary excitations, are quasiparticles of spin  $1/2$ . The hallmark of the HS spin chain is the existence of “supermultiplets”, degenerate energy eigenstates of the same spinon number but different spin. The structure of eigenstates is built on so-called “fully (spin-)polarized spinon gas” (FPSG) states [7] with definite spinon number and maximum spin equal to the half of the spinon number. Their corresponding, degenerate states can be found by acting with an operator of the Yangian algebra, inherent to the model for which the FPSG states are highest weight states. In the case of two spinons we have two degenerate states a triplet and a singlet and the latter wavefunction is  $\Psi'(z_1, \dots, z_{N/2}) = \prod_{j < k}^{N/2} (z_j - z_k)^2 [1 - \prod_{j=1}^{N/2} z_j^2]$ .

*Motivation behind Numerical Calculations* We will present our numerical work later on, but here we will give just a synopsis of what can be established on the basis of the numerical work and how this can be used to prove the existence of merons.

In the case of polarized electrons, QD at  $\nu = 1$  is in a stable state, so-called maximum density droplet (MDD) state, where each angular momentum orbital till its maximum value  $N - 1$  is filled. Therefore its total angular momentum is  $M = N(N - 1)/2$ . In this system the existence of vortex (dressed hole) quasiparticle excitation is firmly established [9, 10, 11]. As we slowly increase the magnetic field (from the MDD value) a particle-hole vortex excitation is formed, where the vortex occupies an inner orbital of the MDD state and at the same time pushes charge outside and creates a particle on the edge. This process is followed by gradual increase of  $M$ . If QD is small enough this description may persist to the point when the vortex quasiparticle is created at the center which can be associated with the total increase of  $\Delta M = N$  of the angular momentum as implied by the shift register counting of Laughlin [12], or, therefore, an increase of one flux quantum in the magnetic flux through the system. Therefore the lowest lying excitations of the dot as the magnetic field is increased can be described through vortex excitations which makes the vortex an eigenstate of the polarized system.

We will show that a similar process happens with unpolarized electrons, starting from the MDD configuration [13], where the quasiparticle that slowly sweeps the inner orbitals, pushes charge outside, and finally is created at the center is meron. Therefore even in this case we will recover usual quasiparticle-hole phenomenology of a QH system and therefore prove the existence of merons. The meron is half of skyrmion, and to skyrmion an increase of one flux quantum is associated because skyrmions are analogs of hole (vortex) excitations in the limit of weak

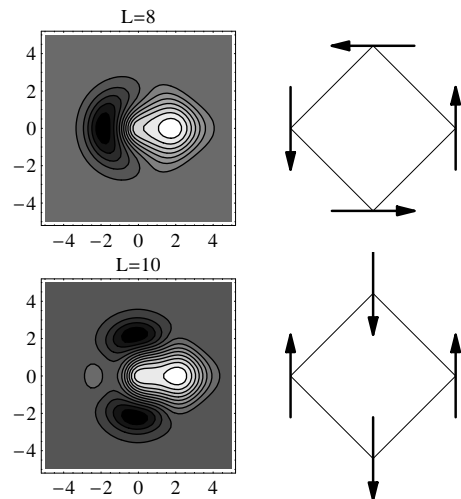


FIG. 2: Spin-spin correlations along fixed radii of lowest lying, spin-singlet states of the  $N = 4$  dot. The reference points are on the positive side of  $x$  axis. On the rhs are two possible classical meron configurations where only the characteristic spin windings along the meron edge are shown.

Zeeman coupling [4]. Therefore if we create a meron at the center we expect half flux quantum increase in the flux through the system and associated increase of  $\Delta M = N/2$  in the total angular momentum. That should be also period for which we expect appearance of merons of higher winding number in very small dots; see  $N = 4$  example below.

Our findings support that the MDD state, with respect to its spin content, can be viewed as a condensate of  $N$  merons spin  $\frac{1}{2}$  where each two merons pair to one hole (a vortex) [14]. We will call these quasiparticle (not quasihole) kind of merons condensate merons. We find, while establishing a mapping between the dot and the HS chain that the ferromagnetically ordered MDD state corresponds to the  $2S = N$ , maximum number of spinons, HS  $N$  site spin chain state. This state, except for the spin degeneracy, is a unique state of the chain. When a quasiparticle, more precisely a meron quasihole, enters the dot, the number of condensate merons is reduced, by creation of a quasiparticle-hole pair, to  $N - 2$ . We find that as an effect the successive quasihole orbital states as it enters a dot can be associated to half of (see [15]) the  $N - 2$  spinon sector of the HS  $N$  site spin chain. Therefore merons map to spinons and we are establishing merons as eigenstates of dot problem [16].

*$N = 4$  dot* In Fig. 2 we show spin-spin correlations along fixed radii, of lowest lying, at fixed  $L$ , spin singlet states. On the rhs of Fig. 2 shown are two possible types of (classical) configurations, windings of merons at their edges projected to four sites. The occurrence of black regions correspond to our expectations where to find antiferromagnetic correlations according to the classical configu-

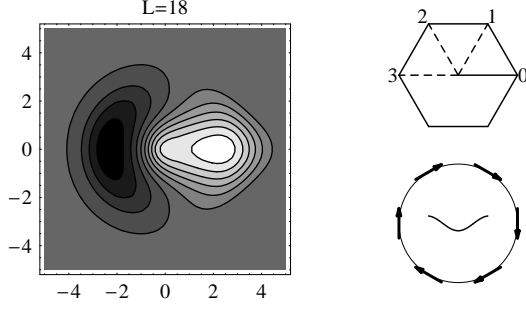


FIG. 3: The map of the spin-spin correlations of the lowest lying, spin-singlet state at  $L = 18$  of the  $N = 6$  dot. In the lower right corner is the classical meron edge configuration for comparison.

rations at some radii, which turn out to be near or at the maximum (radial) density radius. Similar results already appeared in [17] but not with emphasis on the  $\frac{N}{2}$  periodicity (the  $L$  of MDD is at  $L = \frac{N(N-1)}{2} = 6$ ), and meron interpretation we are pointing out here. And indeed after a detailed investigation we found also  $RVB^+$  and  $RVB^-$  four site spin states [17] to describe these states,  $L = 10$  and  $L = 8$  respectively, but in addition we identified these states with the ground and two spinon lowest lying singlet state of the HS four site spin chain respectively. Moreover at  $L = 8$  we identified, with respect to the lowest lying, a nearly degenerate  $S = 1$  state with the triplet,  $z^2$  state of the HS, and this is in the accordance with the mapping to the two degenerate states, a singlet ( $RVB^-$ ) and a triplet ( $z^2$ ) of the  $N - 2 = 2$  spinon sector of the HS chain. Also at  $L = 7$  we found the triplet  $z^3$  two spinon state of the chain as expected [7].

$N=6$  dot Also in the case of the  $N = 6$  dot we will recover meron quasiparticle-hole states as the lowest lying ones for angular momenta higher than, but still near, the MDD state. First we can notice in Fig. 3 at  $\Delta M = N/2 = 3$ ,  $M = 18$  a meron quasihole created at the center of the dot. Before reaching that state as the magnetic field is gradually increased from the MDD state the system passes through the states at  $M = 16$  and  $M = 17$  analyzed in Fig. 4. As announced we expect that the  $N - 2 = 4$  spinon sector of the HS chain is associated with the states at  $M = 16, 17$ , and 18. And indeed just above  $L = \frac{N(N-1)}{2} = 15$  at  $L = 16$  and  $L = 17$  we easily identified these HS states as in Fig. 4 where corresponding states are noted. Plotted are ratios of calculated real parts of  $S^+S^-$  correlations as functions of radius of the dot states subtracted by the same ratios for the HS  $N = 6$  chain for expected HS states. We take ratios because of different electron densities at different dot radii. In general we define the quantities displayed

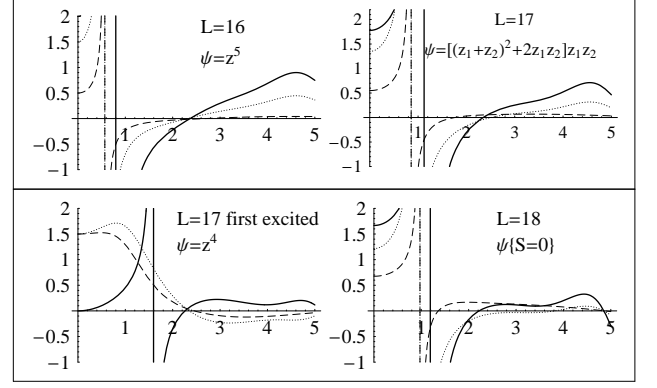


FIG. 4: Quantities  $f_{12}$  - full,  $f_{13}$  - dotted, and  $f_{23}$  - dashed, for the definitions see the text, as functions of radius. The quantities should go to zero, simultaneously if the corresponding HS state, in the Figure denoted for each angular momentum, for  $L = 18$  see the text, describes the dot state correlations along a fixed radius. (Asymptotes are also shown.)

in Fig. 4 as

$$f_{ab} = \frac{Re\langle S^+(r,0)S^-(r,a) \rangle}{Re\langle S^+(r,0)S^-(r,b) \rangle} - \frac{Re\langle S^+(0)S^-(a) \rangle|_{HS}}{Re\langle S^+(0)S^-(b) \rangle|_{HS}}, \quad (5)$$

where  $a$  and  $b$  take values 1,2, and 3 denoting the angles, with respect to the positive  $x$ -axis, represented on the hexagon in the upper right corner of Fig.3. Other HS states with respect to the shown do not show the nice confluences of all three lines,  $f_{12}, f_{13}$ , and  $f_{23}$ , at their simultaneous value zero at a single radius; they are completely off and uncorrelated. Similarly we can define quantities with the imaginary parts of  $S^+S^-$  correlations. Some of them are identically zero, due to the same real correlation property of HS and QD states, and the rest are compatible to and very suggestive of the trend around the special radius that exists in real parts (Fig. 4). That again is the behavior we see only for these special HS states. We find that the HS states' conditions, including  $S_z S_z$  correlations, are satisfied at the radius (Fig. 4) slightly beyond the maximum density radius.

For  $L = 17$  we found the  $S = 2$ ,  $z^4$  closely lying, first excited state that belongs to the expected [7] HS two state degenerate supermultiplet (Figs. 4 and 5.). We emphasize that although the results of the  $N = 4$  case may be justified or expected because of early (for small  $L$ ) Wigner crystal formation [18], we checked that for  $L = 15 - 18$ , in the  $N = 6$  case there is no underlying Wigner crystal configuration [19]. The mapping does not require underlying crystal structure, and may persist in dots with  $N$  higher than 6.

In Fig. 5 we see clearly, beside the  $L = 17$  multiplet, the existence of an additional multiplet at  $L = 18$ , with three states, that we expect [7] if we identify the  $S = 0$  state as the lowest lying  $S = 0$  state in the 4-spinon

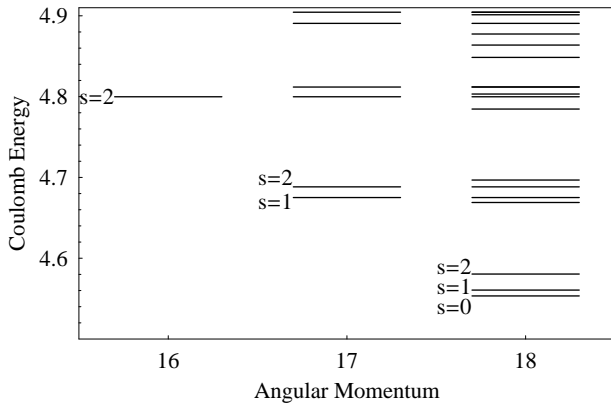


FIG. 5: The spectrum of the  $N = 6$  dot as a function of angular momentum. Lowest energy, closely lying levels correspond to HS supermultiplets [7].

sector,

$$\Psi_{S=0} = \prod_{i<j}^M (z_i - z_j)^2 \prod_i^M z_i + 15 \left\{ \prod_i^M z_i + \prod_i^M z_i^3 + \prod_i^M z_i^5 \right\}, \quad (6)$$

where  $M = 3$ . And indeed we were able to do that, the analysis is given in Fig. 3, where we see a clear correspondence between the classical meron edge configuration ( $2\pi$  winding of the spin vector in the plane) and the spin-spin correlation map, and in Fig. 4, but less easily than the lower momentum states. The reason is that we are already approaching a transition region to the Wigner regime [20, 21] where the Wigner structure with a pentagon and an electron in the middle competes with the hexagonal structure [22]. Therefore the transition region may well consist of liquid states of merons similarly to what happens in the polarized case with vortices [11].

*Final Remarks* The physics of exchange [4, 6] that favors smooth variations of spin in space, favors meron solutions instead of skyrmion ones in small systems, in the limit of zero Zeeman coupling. Merons distort spin at slower rate than skyrmions, although acquire also energy due to winding over the space of the system. This second contribution is suppressed in small systems [23] where merons, as we demonstrated here, constitute lowest lying eigenstates.

The meron physics should be detectable in lateral quantum dots, in which interaction effects are strong, in their not yet explored regime beyond the MDD state [24]. The same conclusions are valid for rapidly rotating fermi atoms, where there is no Zeeman effect to disguise the fractionalization into merons, and, therefore, the RVB and spinon physics implied by the mapping to the HS chain may reveal more easily.

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  - [13] Here we use the notion of the MDD state not as of a maximum possible density state, but as the one in which in the orbital space, each orbital till the value  $N - 1$  is filled by exactly one electron.
  - [14] We noticed the presence of vortex excitations in the low lying spectrum.
  - [15] As may be expected only one chirality (angular momentum direction) branch maps to the states of QD in a magnetic field.
  - [16] It is not surprising that the mapping we find is to the HS chain. The chain is a paradigm of quantum antiferromagnet fractionalization and its excitations, spinons, have semionic statistics. Merons as halves of skyrmions, which at  $\nu = 1$  carry fermionic statistics, are expected to have semionic statistics. In principle it is hard to talk about statistics in small systems but if we have signatures of statistics in small chains [7], to which mapping is possible, they should carry information of the expected quasiparticle statistics in QDs.
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